Late-Time Evolution of Charged Gravitational Collapse and Decay of Charged Scalar Hair - II

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Abstract

We study *analytically* the initial value problem for a *charged* massless scalar-field on a Reissner-Nordström spacetime. Using the technique of spectral decomposition we extend recent results on this problem. Following the no-hair theorem we reveal the *dynamical* physical mechanism by which the *charged* hair is radiated away. We show that the charged perturbations decay according to an inverse power-law behaviour at future timelike infinity and along future null infinity. Along the future outer horizon we find an oscillatory inverse power-law relaxation of the charged fields. We find that a *charged* black hole becomes "bald" *slower* than a neutral one, due to the existence of charged perturbations. Our results are also important to the study of mass-inflation and the stability of Cauchy horizons during a dynamical gravitational collapse of charged matter in which a charged black-hole is formed.

I. INTRODUCTION

The late-time evolution of various fields outside a collapsing star plays an important role in two major aspects of black-hole physics:

1. The *no-hair theorem* of Wheeler states that the *external* field of a black-hole relaxes to a Kerr-Newman field characterized solely by the black-hole's mass, charge and angular-

momentum. Thus, it is of interest to reveal the *dynamical* mechanism responsible for the relaxation of perturbations fields outside a black-hole. The mechanism by which neutral fields are radiated away was first studied by Price [1]. The physical mechanism by which a charged black-hole, which is formed during a gravitational collapse of a *charged* matter, dynamically sheds its charged hair was first studied in paper I [2]. However, this analysis was restricted to the weak electromagnetic interaction limit $|eQ| \ll 1$. In this paper we extend our *analytical* results to include *general* values of the (dimensionless) quantity eQ.

2. The asymptotic late-time tails along the outer horizon of a rotating or a charged black-hole are used as initial input for the ingoing perturbations which penetrates into the black-hole. These perturbations are the physical cause for the well-known phenomena of *mass-inflation* [3]. In this context, one should take into account the existence of *charged* perturbations, which are expected to appear in a dynamical gravitational collapse of a *charged* star. Here we study the asymptotic behaviour of such charged perturbations.

The plan of the paper is as follows. In Sec. II we give a short description of the physical system and formulate the evolution equation considered. In Sec. III we formulate the problem in terms of the black-hole Green's function using the technique of spectral decomposition. In Sec. IV we study the late-time evolution of charged scalar perturbations on a Reissner-Nordström background. We find an inverse *power-law* behaviour of the perturbations along the three asymptotic regions: timelike infinity i_+, future null infinity $scri_+$ and along the black-hole outer-horizon (where the power-law is multiplied by a *periodic* term). We find that the dumping exponents which describe the fall-off of *charged* perturbations are *smaller* compared with the neutral dumping exponents. Thus, a black-hole which is formed from the gravitational collapse of a *charged* matter becomes "bald" *slower* than a neutral one due to the existence of charged perturbations. In Sec. V we reduce our results to the Schwarzschild case (and equivalently, to a neutral field on a Reissner-Nordström back-

ground). We show that one can obtain the asymptotic behaviour of the field along the outer horizon [4] using the technique of spectral decomposition. We conclude in Sec. VI with a brief summary of our results and their implications.

II. DESCRIPTION OF THE SYSTEM

We consider the evolution of a massless *charged* scalar perturbations fields outside a charged collapsing star. The system was already described on paper I [2]. Here we give only the final form of the equations studied. The external gravitational field of a spherically symmetric collapsing star of mass M and charge Q is given by the metric

$$ds^2 = \lambda^2(-dt^2 + dy^2) + r^2 d\Omega^2 \ , \tag{1}$$

where the tortoise radial coordinate y is defined by $dy = dr/\lambda^2$ and $\lambda^2 = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$.

Resolving the field into spherical harmonics $\phi = e^{ie\Phi t} \sum_{l,m} \psi^l_m \left(t, r\right) Y^m_l(\theta, \varphi)/r$ (where Φ is merely a gauge constant of the electromagnetic potential A_t, i.e. its value at infinity) one obtains a wave- equation for each multipole moment

$$\psi_{,tt} + 2ie\frac{Q}{r}\psi_{,t} - \psi_{,yy} + V\psi = 0 \ , \tag{2}$$

where

$$V = V_{M,Q,l,e}\left(r\right) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\left[\frac{l\left(l+1\right)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4}\right] - e^2\frac{Q^2}{r^2} \ . \tag{3}$$

The electromagnetic potential A_t satisfies the relation

$$A_t = \Phi - \frac{Q}{r} \ , \tag{4}$$

where Φ is a constant.

III. FORMALISM

The time-evolution of a charged scalar- field described by Eq. (2) is given by

$$\psi(y,t) = \int \left[G(y,x;t)\psi_t(x,0) + G_t(y,x;t)\psi(x,0) + \frac{2ieQ}{r(x)} G(y,x;t)\psi(x,0) \right] dx \ , \tag{5}$$

for $t > 0$, where the (retarded) Green's function $G(y,x;t)$ is defined as

$$\left[\frac{\partial^2}{\partial t^2} + 2ie\frac{Q}{r}\frac{\partial}{\partial t} - \frac{\partial^2}{\partial y^2} + V(r) \right] G(y,x;t) = \delta(t)\delta(y-x) \ . \tag{6}$$

The causality condition gives us the initial condition $G(y,x;t) = 0$ for $t \leq 0$. In order to find $G(y,x;t)$ we use the Fourier transform

$$\tilde{G}(y,x;w) = \int_{0^-}^{\infty} G(y,x;t)e^{iwt}dt \ . \tag{7}$$

The Fourier transform is analytic in the upper half w-plane and it satisfies the equation

$$\left(\frac{d^2}{dy^2} + w^2 - \frac{2eQw}{r} - V \right) \tilde{G}(y,x;w) = \delta(y-x) \ . \tag{8}$$

$G(y,x;t)$ itself is given by the inversion formula

$$G(y,x;t) = \frac{1}{2\pi} \int_{-\infty+ic}^{\infty+ic} \tilde{G}(y,x;w)e^{-iwt}dw \ , \tag{9}$$

where c is some positive constant.

Next, we define two auxiliary functions $\tilde{\psi}_1(y,w)$ and $\tilde{\psi}_2(y,w)$ which are (linearly independent) solutions to the homogeneous equation

$$\left(\frac{d^2}{dy^2} + w^2 - \frac{2eQw}{r} - V \right) \tilde{\psi}_i(y,w) = 0 \ , \qquad i = 1,2 \ . \tag{10}$$

The two basic solutions that are required in order to build the black-hole Green's function are defined by their asymptotic behaviour:

$$\tilde{\psi}_1(y,w) \sim \begin{cases} e^{-i\left(w-\frac{eQ}{r_+}\right)y} & , \quad y \to -\infty \ , \\ A_{out}(w)y^{i(2wM-eQ)}e^{iwy} + A_{in}(w)y^{-i(2wM-eQ)}e^{-iwy} & , \quad y \to \ \infty \ , \end{cases} \tag{11}$$

and

$$\tilde{\psi}_2(y,w) \sim \begin{cases} B_{out}(w)e^{i\left(w-\frac{eQ}{r_+}\right)y} + B_{in}(w)e^{-i\left(w-\frac{eQ}{r_+}\right)y} & , \quad y \to -\infty \ , \\ y^{i(2wM-eQ)}e^{iwy} & , \quad y \to \infty \ . \end{cases} \tag{12}$$

Let the Wronskian be

$$W(w) = W(\tilde{\psi}_1, \tilde{\psi}_2) = \tilde{\psi}_1 \tilde{\psi}_{2,y} - \tilde{\psi}_2 \tilde{\psi}_{1,y} \ , \tag{13}$$

where $W(w)$ is y-independent. Thus, using the two solutions $\tilde{\psi}_1$ and $\tilde{\psi}_2$, the black- hole Green's function can be expressed as

$$\tilde{G}(y, x; w) = -\frac{1}{W(w)} \begin{cases} \tilde{\psi}_1(y, w)\tilde{\psi}_2(x, w) & , \quad y < x \ , \\ \tilde{\psi}_1(x, w)\tilde{\psi}_2(y, w) & , \quad y > x \ . \end{cases} \tag{14}$$

In order to calculate $G(y, x; t)$ using Eq. (9), one may close the contour of integration into the lower half of the complex frequency plane. Then, one finds three distinct contributions to $G(y, x; t)$ [5] :

1. *Prompt contribution.* This comes from the integral along the large semi-circle. It is this part, denoted G^F, which propagates the *high*- frequency response. For large frequencies the Green's function limits to the flat spacetime one. Thus, this term contributes to the *short*-time response (radiation which reaches the observer nearly directly from the source, i.e. without scattering). This term can be shown to be effectively zero beyond a certain time. Thus, it is not relevant for the late-time behaviour of the field.

2. *Quasinormal modes.* This comes from the distinct singularities of $\tilde{G}(y, x; w)$ in the lower half of the complex w-plane and is denoted by G^Q. These singularities occur at frequencies for which the Wronskian (13) vanishes. G^Q is just the sum of the residues at the poles of $\tilde{G}(y, x; w)$. Since each mode has $\mathrm{Im}w < 0$ it decays *exponentially* with time.

3. *Tail contribution.* The late-time tail is associated with the existence of a branch cut (in $\tilde{\psi}_2$) [5], usually placed along the negative imaginary w-axis. This tail arises from the integral of $\tilde{G}(y, x; w)$ around the branch cut and is denoted by G^C. As will be shown, the contribution G^C leads to an inverse *power-law* behaviour (multiplied by a periodic term along the black-hole outer horizon) of the field. Thus, G^C dominates the late-time behaviour of the field.

The present paper investigate the late-time asymptotic behaviour of a charged scalar-field. Thus, the purpose of this paper is to evaluate $G^C(y,x;t)$.

IV. THE LATE-TIME BEHAVIOUR OF A CHARGED SCALAR-FIELD

A. The large-r (low-w) approximation

It is well known that the late-time behaviour of massless perturbations fields is determined by the backscattering from asymptotically *far* regions [6,1]. Thus, the late-time behaviour is dominated by the *low*-frequencies contribution to the Green's function, for only low frequencies will be backscattered by the small potential (for $r \gg M, |Q|$) in (10). Thus, as long as the observer is situated far away from the black-hole and the initial data has a considerable support only far away from the black-hole, a *large-r* (or equivalently, a *low-w*) approximation is sufficient in order to study the asymptotic *late-time* behaviour of the field [7].

The wave-equation (10) of the charged scalar-field (in the field of a charged black hole) can be transformed in such a way that the Coulomb and Newtonian $1/r$ potentials will dominate at large values of r. We first introduce an auxiliary field ξ

$$\xi = \lambda \tilde{\psi} \ , \tag{15}$$

in terms of which the equation (10) for the charged scalar-field becomes

$$\left\{ \frac{d^2}{dr^2} - \frac{\lambda_{,rr}}{\lambda} + \frac{1}{\lambda^4} \left[\left(w - \frac{eQ}{r} \right)^2 - \lambda^2 \left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} \right) \right] \right\} \xi = 0 \ . \tag{16}$$

As was explained above, one only needs a *large* r approximation in order to study the late-time behaviour of the charged field. Thus, we expend (16) as a power series in M/r and Q/r and obtain (neglecting terms of order $O(\frac{w}{r^2})$ and smaller)

$$\left[\frac{d^2}{dr^2} + w^2 + \frac{4Mw^2 - 2eQw}{r} - \frac{l(l+1) - (eQ)^2}{r^2} \right] \xi = 0 \ . \tag{17}$$

The terms proportional to M/r and eQ/r represent the Newtonian and Coulomb potentials respective. It should be noted that this equation (with $M = 0$) represents *exactly* the evolution of a *charged* scalar-field on a *charged* and *flat* background.

Let us now introduce a second auxiliary field $\tilde{\phi}$ defined by

$$\xi = r^{\beta+1} e^{iwr} \tilde{\phi}(z) \ , \tag{18}$$

where

$$z = -2iwr \quad ; \quad \beta = \frac{-1 + \sqrt{(2l+1)^2 - 4(eQ)^2}}{2} \ . \tag{19}$$

$\tilde{\phi}$(z) satisfies the confluent hypergeometric equation

$$\left[z\frac{d^2}{dz^2} + (2\beta + 2 - z)\frac{d}{dz} - (\beta + 1 - 2iw\alpha) \right] \tilde{\phi}(z) = 0 \ , \tag{20}$$

where

$$\alpha = M - \frac{eQ}{2w} \ . \tag{21}$$

Thus, the two basic solutions required in order to build the black-hole Green's function are (for $r \gg M, |Q|$)

$$\tilde{\psi}_1 = Ar^{\beta+1} e^{iwr} M(\beta + 1 - 2iw\alpha, 2\beta + 2, -2iwr) \ , \tag{22}$$

and

$$\tilde{\psi}_2 = Br^{\beta+1} e^{iwr} U(\beta + 1 - 2iw\alpha, 2\beta + 2, -2iwr) \ , \tag{23}$$

where A and B are normalization constants. $M(a, b, z)$ and $U(a, b, z)$ are the two standard solutions to the confluent hypergeometric equation [8]. $U(a, b, z)$ is a many-valued function, i.e. there will be a cut in $\tilde{\psi}_2$.

Using Eq. (9), one finds that the branch cut contribution to the Green's function is given by

$$G^C(y, x; t) = \frac{1}{2\pi} \int_0^{-i\infty} \tilde{\psi}_1(x, w) \left[\frac{\tilde{\psi}_2(y, we^{2\pi i})}{W(we^{2\pi i})} - \frac{\tilde{\psi}_2(y, w)}{W(w)} \right] e^{-iwt} dw \ . \tag{24}$$

(For simplicity we assume that the initial data has a considerable support only inside the observer. This, of course, does not change the *late*-time behaviour, for it is a consequence of a backscattering at asymptotically *far* regions).

Using the fact that $M(a,b,z)$ is a single-valued function and Eq. 13.1.10 of [8], one finds

$$\tilde{\psi}_1(r, we^{2\pi i}) = \tilde{\psi}_1(r, w) \ , \tag{25}$$

and

$$\tilde{\psi}_2(r, we^{2\pi i}) = e^{-4i\pi\beta}\tilde{\psi}_2(r, w) + (1 - e^{-4i\pi\beta})\frac{\Gamma(-2\beta-1)}{\Gamma(-\beta-2iw\alpha)}\tilde{\psi}_1(r, w) \ . \tag{26}$$

Using Eqs. (25) and (26) it is easy to see that

$$W(we^{2\pi i}) = W(w) \ . \tag{27}$$

Thus, using Eqs. (25),(26) and (27), we obtain the relation

$$\frac{\tilde{\psi}_2(r, we^{2\pi i})}{W(we^{2\pi i})} - \frac{\tilde{\psi}_2(r, w)}{W(w)} = \frac{B}{A}\left(e^{4\pi i\beta} - 1\right)\frac{\Gamma(-2\beta-1)}{\Gamma(-\beta-2iw\alpha)}\frac{\tilde{\psi}_1(r, w)}{W(w)}. \tag{28}$$

Since $W(w)$ is r-independent, we may use the large-r asymptotic expansions of the confluent hypergeometric functions (given by Eqs. 13.5.1 and 13.5.2 in [8]) in order to evaluate it. One finds

$$W(w) = -i\frac{AB\Gamma(2\beta+2)e^{\pi i\beta}w^{-2\beta-1}}{\Gamma(\beta+1-2iw\alpha)2^{2\beta+1}} \ . \tag{29}$$

(Of coarse, using the $|z| \to 0$ limit of the confluent hypergeometric functions, we obtain the same result). Thus, substituting (28) and (29) in (24) we obtain

$$G^C(y, x; t) = \frac{i2^{2\beta}\Gamma(-2\beta-1)\left(e^{3\pi i\beta} - e^{-\pi i\beta}\right)}{\pi A^2\Gamma(2\beta+2)}\int_0^{-i\infty}\frac{\Gamma(\beta+1-2iw\alpha)}{\Gamma(-\beta-2iw\alpha)}w^{2\beta+1}\tilde{\psi}_1(y, w)\tilde{\psi}_1(x, w)e^{-iwt}dw \ . \tag{30}$$

B. Asymptotic behaviour at timelike infinity

As was explained, the late-time behaviour of the field should follow from the *low*-frequency contribution to the Green's function. Actually, it is easy to verify that the effective contribution to the integral in (30) should come from $|w|=O(\frac{1}{t})$. Thus, in order to obtain the asymptotic behaviour of the field at *timelike infinity* i_+ (where $x, y \ll t$), we may use the $|w|r \ll 1$ limit of $\tilde{\psi}_1(r,w)$. Using Eq. 13.5.5 from [8] one finds

$$\tilde{\psi}_1(r,w) \simeq Ar^{\beta+1} \ . \tag{31}$$

Thus, we obtain

$$G^C(y,x;t) = \frac{i2^{2\beta}\Gamma(-2\beta-1)\Gamma(\beta+1+ieQ)(e^{3\pi i\beta}-e^{-\pi i\beta})}{\pi\Gamma(2\beta+2)\Gamma(-\beta+ieQ)}(yx)^{\beta+1}\int_0^{-i\infty} w^{2\beta+1}e^{-iwt}dw \ , \tag{32}$$

where we have used the relation $-2iw\alpha \simeq ieQ$ for $w \to 0$. Performing the integration in (32), one finds that

$$G^C(y,x;t) = \frac{\Gamma(-2\beta-1)\Gamma(\beta+1+ieQ)2^{2\beta+1}sin(2\pi\beta)}{\pi\Gamma(-\beta+ieQ)}x^{\beta+1}y^{\beta+1}t^{-(2\beta+2)} \ . \tag{33}$$

Thus, the late-time behaviour of the *charged* scalar-field is dominated by the *electromagnetic* interaction (a *flat* spacetime effect) rather then by the spacetime curvature. Moreover, we should point out an interesting and unique feature of this result. Contrary to neutral perturbations, where the amplitude of the late-time tail is *proportional* to the curvature of the spacetime (to M), the late-time behaviour of the *charged* scalar-field is *not* linear in the electromagnetic interaction (in the quantity eQ). In other words, the first Born approximation is *not* valid for general values of the quantity eQ (in the first Born approximation, the amplitude to be backscattered and thus the late-time field itself, are *linear* in the scattering potential). The *physical* significance of this result is the fact that the late-time behaviour of the *charged* scalar-field is dominated by *multiple* scattering from asymptotically far regions. This is exactly the physical reason responsible for the fact that the analysis given in [2] is restricted to the $|eQ| \ll 1$ case (where the first Born approximation is valid).

C. Asymptotic behaviour at future null infinity

Next, we go on to consider the behaviour of the charged scalar-field at future null infinity $scri_+$. It is easy to verify that for this case the effective frequencies contributing to integral (30) are of order $O(\frac{1}{u})$. Thus, for $y-x \ll t \ll 2y-x$ one may use the $|w|x \ll 1$ asymptotic limit for $\tilde{\psi}_1(x,w)$ and the $|w|y \gg 1$ $(Imw < 0)$ asymptotic limit of $\tilde{\psi}_1(y,w)$. Thus,

$$\tilde{\psi}_1(x,w) \simeq Ax^{\beta+1} \ , \tag{34}$$

and

$$\tilde{\psi}_1(y,w) \simeq Ae^{iwy}\Gamma(2\beta+2)\frac{e^{-i\frac{\pi}{2}(\beta+1-2iw\alpha)}(2w)^{-\beta-1+2iw\alpha}y^{2iw\alpha}}{\Gamma(\beta+1+2iw\alpha)} \ , \tag{35}$$

where we have used Eqs. 13.5.5 and 13.5.1 of [8], respectively. Using this low-frequency limit one finds (for $v \gg u$)

$$G^C(y,x;t) = \frac{\Gamma(-2\beta-1)\Gamma(\beta+1+ieQ)2^{\beta}sin(2\pi\beta)}{\pi\Gamma(-\beta+ieQ)}x^{\beta+1}v^{-ieQ}u^{-(\beta+1-ieQ)} \ . \tag{36}$$

D. Asymptotic behaviour along the black-hole outer horizon

Finally, we consider the behaviour of the charged scalar-field at the black-hole outer-horizon r_+. While (22) and (23) are (approximated) solutions to the wave-equation (10) in the $r \gg M, |Q|$ case, they do not represent the solution near the horizon. As $y \to -\infty$ the wave-equation (10) can be approximated by the equation

$$\tilde{\psi}_{,yy} + \left(w - \frac{eQ}{r_+}\right)^2 \tilde{\psi} = 0 \ . \tag{37}$$

Thus, we take

$$\tilde{\psi}_1(y,w) = C(w)e^{-i\left(w-\frac{eQ}{r_+}\right)y} \ , \tag{38}$$

and we use (34) for $\tilde{\psi}_1(x,w)$. In order to match the $y \ll -M$ solution with the $y \gg M$ solution we assume that the two solutions have the same temporal dependence (this

assumption has been proven to be very successful for neutral perturbations [4]). In other words we take $C(w)$ to be w-independent. In this case one should replace the roles of x and y in Eqs. (24) and (30). Using (30), we obtain

$$G^C(y,x;t) = \Gamma_0 \frac{\Gamma(-2\beta-1)\Gamma(\beta+1+ieQ)2^{2\beta+1}sin(2\pi\beta)}{\pi\Gamma(-\beta+ieQ)} e^{i\frac{eQ}{r_+}y} v^{-(2\beta+2)} \ , \tag{39}$$

where Γ_0 is a constant.

E. The $|eQ| \ll 1$ case

Let us compare the results obtained in this paper with those obtained in [2], using a *different* approach (for $|eQ| \ll 1$). Taking the $|eQ| \ll 1$ limit of Eqs. (33),(36) and (39) one finds

$$G^C(y,x;t) = \frac{2ieQ(-1)^l(2l)!!}{(2l+1)!!} x^{\beta+1} y^{\beta+1} t^{-(2\beta+2)} \ , \tag{40}$$

at timelike infinity i_+,

$$G^C(y,x;t) = \frac{ieQ(-1)^l l!}{(2l+1)!!} x^{\beta+1} v^{-ieQ} u^{-(\beta+1-ieQ)} \ , \tag{41}$$

at future null infinity $scri_+$ and

$$G^C(y,x;t) = \Gamma_0 \frac{2ieQ(-1)^l(2l)!!}{(2l+1)!!} e^{i\frac{eQ}{r_+}y} v^{-(2\beta+2)} \ , \tag{42}$$

along the black-hole outer-horizon r_+, respectively. These results have exactly the same temporal and spatial dependence as those obtained in [2] for the $|eQ| \ll 1$ case (and for $v, t \ll |Q| e^{\frac{1}{|eQ|}}$).

V. THE SCHWARZSCHILD BLACK-HOLE

For a Schwarzschild black-hole ($Q=0, M \neq 0$) we have $\alpha = M$ and $\beta = l$. Thus, in this case, expression (30) reduces to (taking $M|w| \to 0$)

$$G^C(y,x;t) = \frac{4iM}{A^2\left[(2l+1)!!\right]^2} \int_0^{-i\infty} w^{2l+2} \tilde{\psi}_1(y,w) \tilde{\psi}_1(x,w) e^{-iwt} dw \ . \tag{43}$$

Now, taking the appropriate approximations for $\tilde{\psi}_1(y,w)$ and $\tilde{\psi}_1(x,w)$ (as is done in Sec. IV), one finds

$$G^C(y,x;t) = \frac{(-1)^{l+1} 4M(2l+2)!}{[(2l+1)!!]^2} x^{l+1} y^{l+1} t^{-(2l+3)} \ , \tag{44}$$

at timelike infinity i_+,

$$G^C(y,x;t) = \frac{(-1)^{l+1} 2M(2l+2)!}{(2l+1)!!} x^{l+1} u^{-(l+2)} \ , \tag{45}$$

at future null infinity $scri_+$, and

$$G^C(y,x;t) = \Gamma_1 \frac{(-1)^{l+1} 4M(2l+2)!}{[(2l+1)!!]^2} x^{l+1} v^{-(2l+3)} \ , \tag{46}$$

along the black-hole outer horizon r_+, where Γ_1 is a constant. Result (44) is just the one obtained by Leaver [5] and subsequently by the simplified approach of [7]. Result (45) is the one obtained in [5]. Thus, the simplified approach of [7] can be *extended* to include also the asymptotic behaviour at future null infinity and along the future outer horizon. Furthermore, result (46) is new in the sense that it gives the $v^{-(2l+3)}$ dependence of the field along the black-hole outer horizon using a *different* approach compared with the one given in [4]. Obviously, results (44),(45) and (46) are also valid for the case of a *neutral* scalar-field, evolved on a Reissner-Nordström background, for then one also has $\alpha = M$ and $\beta = l$.

VI. SUMMARY AND PHYSICAL IMPLICATIONS

We have studied the asymptotic late-time behaviour of a *charged* gravitational collapse. Following the *no-hair theorem* we have focused attention on the physical mechanism by which a *charged* hair is radiated away. The main results and their physical implications are:

1. Inverse *power-law* tails develop at timelike infinity, at null infinity and along the black-hole outer horizon (where the power-law behaviour is multiplied by an oscillatory term).

2. The dumping exponents for the *charged* field are *smaller* compared with those of neutral perturbations. This implies that a black-hole which is formed from a gravitational collapse of a charged matter becomes "bald" *slower* than a neutral one (due to the existence of charged perturbations).

3. Since *charged* perturbations decay as a *power-law* they are expected to cause a *mass-inflation* singularity during a charged gravitational collapse which leads to the formation of a charged black-hole. Moreover, since charged perturbations decay *slower* than neutral ones they are expected to *dominate* the mass-inflation phenomena during a charged gravitational collapse. [This is caused by the fact that the mass-function diverges like $m(v) \simeq v^{-p}e^{\kappa_0}$, where $\frac{1}{2}p$ is the dumping exponent of the field [3].]

4. While the late-time behaviour of neutral perturbations is dominated by the spacetime *curvature*, the late-time behaviour of *charged* fields is dominated by *flat* spacetime effects, namely by the *electromagnetic interaction*.

5. While the amplitude of neutral tails is proportional to the the spacetime curvature (to M), the amplitude of *charged* tails is *not* linear in the electromagnetic interaction (in the quantity eQ). This means that the late-time behaviour of *charged* perturbations is physically determined by *multiple* scatterings (contrasted with neutral perturbations). Thus, a first Born approximation which is valid in the case of neutral perturbations is *not* valid for *charged* ones. This is the physical reason for which the analysis of paper I [2] is restricted to the $|eQ| \ll 1$ case.

6. The results given in this paper can be reduced easily to the case of a Schwarzschild black-hole (or a neutral field on a Reissner-Nordström background). For this case we demonstrate that one can obtain the asymptotic behaviour of the field along the black-hole outer horizon using the technique of spectral decomposition.

In accompanying forthcoming papers we study the *fully nonlinear* gravitational collapse of a charged matter. We confirm *numerically* our *analytical* predictions in the case of a

dynamically charged gravitational collapse which leads to a formation of a charged black-hole.

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